The Equation for Excellence

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The Equation For Excellence

How to Make Your Child Excel at Math

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This book is dedicated to my mother, who taught me to excel,
and to my father, who taught me to think independently.

CONTENTS

In this text, the pronoun "he" should be understood to mean "he or she."
The methods discussed apply to both boys and girls.

1: WHY STUDY MATH?

When children ask why they need to study math, the answer usually has something to do with either daily life or applications to science and technology. The problem with the first motivation is that it is an obvious and transparent lie. The second type of "motivation" tends to have the opposite of the intended effect.

The "daily life" explanation tells students that they will need math for their daily activities. For example, they will need to calculate the tip in a restaurant, or determine how much they should pay for their groceries. Most students are quick to point out that this problem can be solved by carrying around a calculator. And anyone who is worried about running out of batteries can carry around a spare set of batteries, or even two calculators. Even cell phones have built in calculators.

The arguments against the "daily life" explanation continue. In daily life, you never need to do more than add, subtract, multiply, or divide. Why learn trigonometry? Why study calculus? Why do anything beyond arithmetic? Even math-oriented jobs rarely require any really advanced math. When I worked as an actuary, the only math I used on the job was multiplication and the occasional exponent (the actuarial profession is one of the most math-oriented professions in the world.)

The other rationale for studying math focuses on science and technology. We need math to design space shuttles and satellites, to work in laboratories, and to build the newest computers. In one way this argument makes sense. Much of that work requires intensive use of advanced math. But very few people work in those areas. Those that work in those areas usually do so because of an internal passion, not because of any external motivation.

In fact, from the perspective of most students, there is very little external motivation to be a scientist. The strongest external motivators for most teenagers are money, fame, power, popularity, and attraction to the opposite sex. None of these powerfully motivate students to pursue careers in science. For every million dollars a scientist makes, the businessmen for whom he works make a billion. For every famous scientist, there are a thousand famous musicians and actors. The scientists who made the nuclear bomb were not the ones to use it; that power belonged to politicians. And in American culture, scientists have no more popularity or sex appeal than anyone else.

Thus, this argument not only fails to motivate students; it actually does the reverse. A student with no interest in being a scientist who hears the technology argument now thinks that advanced math is useful only for scientists. Thus, he does not need to learn it. If his goal is personal gain, his time is better spent doing almost anything else – studying politics, learning to play the guitar, working out, or thinking of ways to make himself rich. Math becomes just an annoying requirement.

So then why should a student learn math at all?

Kings used to play chess to learn military strategy. When I first heard this at age ten, the idea struck me as unbelievably stupid. In chess the bishop can move only diagonally. The knight can move in an L shape. A real soldier, on the other hand, can move in any direction. How would studying chess help in any real war?

I had, of course, completely missed the point. Strategy has nothing to do with L shapes or diagonals. A chess player learns to anticipate his opponent. He learns to look for strong positions, rather than short term gains. He learns to make intelligent sacrifices, and be wary of the strategic artifices of his opponent. He learns to predict his opponent's future responses to his actions, rather than focusing on the immediate gains. This mental discipline makes his mind sharper, and he becomes a much more capable strategist.

Similarly, math is important not because it teaches a student how to use trigonometry to measure the height of a building, but because it develops a student's ability to analyze and solve unfamiliar problems. Math develops concrete reasoning, spatial reasoning, and logical reasoning. Math does not just develop skills that can be applied to science and technology; when math is taught right, it develops the student's fundamental cognitive architecture, increasing his intelligence. The student will develop the logical reasoning skills that allow a lawyer to analyze a legal situation and to present a coherent and convincing argument. He will develop the ability, essential for any businessperson, to isolate the key components of a system. He will develop mental skills that can be used in any problem-solving situation. His mind will become faster, sharper, and more precise.

What lifting weights does for muscles, math does for the mind. In no sport will an athlete suddenly lie down on his back and lift a weight ten times. However, the vast majority of athletes do the press. Why? It makes them stronger, and thus prepares them for athletic endeavors in general.

When you teach a child math in the right way, you are giving him the gift of a sharper and more powerful intelligence. You are helping him actually develop his mind. You are making him smarter. You are giving him the ultimate ability to succeed in the world, and to build a happier life for himself. You are not just making him better at math; you are making him better at thinking.

This book will show you how to make any student excel at math, even a student who is extremely lazy or innately bad at math. You will learn how to motivate any student and what to teach. Whether you are great at math or barely able to do algebra, there are techniques in this book that you can use.

There are a few things you need to know before continuing. The first is that the methods in this book are designed to be effective. They are not designed to be easy, nor are they designed to be fun.

On the flip side, this book does not advocate a "beat your kids to make them strong" type of approach. I never yell at any student, and I obviously do not use any physical punishment. If you do your part right, you will never need to yell at a student to teach him math.

Similarly, the techniques here are not ones designed to cause antagonism. Many of my students spend a good portion of their tutoring sessions frustrated with a math problem, begging for an answer, or literally groaning. And yet the ones who complain the most are the ones who seem to appreciate my training the most. In fact, many of those students pay for part of their tutoring fees with money received from allowances, jobs, and internships, rather than switch to a more moderately priced tutoring service. Instead of spending that money on entertainment, they voluntarily spend it to learn math.

Why would a teenager actually spend his own money to learn math? Because at some level every person desires ability more than entertainment. Although we often believe the opposite, most teenagers would rather gain intelligence than momentary enjoyment. I may make my students struggle more than another educator would. But my methods bring out their very best, and they can see it.

About half of this book focuses on motivation. Right now, some of the finest minds in the country are using every advertising trick they know of to persuade your child to act in certain ways. Alcohol and tobacco companies spend millions of dollars per year on advertising, as do hundreds of junk food, clothing, and entertainment companies. Thus, in today's world, weak motivational methods simply cannot compete. Parents and educators who want to be effective must use motivational methods as powerful as those used by today's professional persuasion artists.

In fact, you will have to be even more persuasive. Unlike an advertiser promoting entertainment or recreation, to effectively teach your child math, you will have to persuade him to take the more difficult, yet ultimately more rewarding, path.

For example, many schools allow students to use calculators. As this book explains, chronic calculator use can dramatically weaken a child's math abilities. Thus, you may be the one persuading your child to not use his calculator, even though his teacher encourages calculator use.

While that task might seem impossible, the methods in this book will show you what to do. Once students understand the damage that calculator use causes, most of them voluntarily stop using calculators altogether. Several of my students have even taken the SAT, the most important test of their lives, without calculators. Almost all of them returned with perfect SAT math scores.

The fact that you will be working to fundamentally improve your child's life will make motivation a bit easier. Even when kids complain, they know what benefits them. And over time, as they see themselves becoming more intelligent and more successful, motivation will become easier.

The rest of this book explains what to teach, and how to teach it. It explores the primary effective math teaching methods, including the legendary Asian system and the methods that underlie the success of my company, Arvin Vohra Education.

2: THE ASIAN SYSTEM

The belief that Asians are good at math is held with good reason: even in America, students with Asian parents tend to significantly outperform every other ethnic group. For example, 2005 math proficiency testing showed that Asian students had higher math proficiency scores than White, Black, and Hispanic students at all age levels *(Source: Child Trends Databank)*.

This section examines the techniques used by Asian parents and educators. Of course, there are variations depending on the country of origin and the individual, but there are techniques and principles that are almost universal among Asian parents and educators.

The Asian system is built on memorization. At an early age, children are taught to memorize multiplication tables and the like. As they get older, they memorize formulas, and even memorize step by step ways to solve specific problem types.

The Asian system is radically different from current American methods, which emphasize understanding over memorization. Where American parents and math teachers focus on explaining why a technique works, the Asian educators simply require that the student memorize the technique, and be ready to use it.

One might expect that such a technique would create students who simply have formulas memorized and are unable to understand what they are doing. But the reality is just the opposite. Once students have the information memorized, the understanding seems to come naturally. On the other hand, systems that drop memorization and focus on only understanding seem to have the reverse effect. Students often end up confused – unable to understand the problem, or to solve it.

This is one of the strangest paradoxes in math education, one that I wrestled with extensively at the beginning of my career as an educator. Why does memorization work in math? Why does focusing exclusively on understanding fail? Isn't math about understanding? Shouldn't memorization be saved for history?

To unravel this mystery, we will undertake a journey that will help us understand some of the most important cognitive principles involved in math education.

COGNITIVE OVERLOAD

Memorize the following list of words:
Cow, dog, horse, tree, sea, frog

Not too hard, right? Now memorize this list:
Frog, moss, grass, house, cow, mouse, deer, phone, well, spoon, table.

The fact that the list is longer makes it much harder. There are various memorization techniques a person can use to memorize the list, but it is not nearly as easy to memorize as the first list.

Most people can hold about 7 pieces of information in their working memory at any given time (usually between 5 and 9 items, depending on their complexity. Working memory is used to remember information for a short period of time. Long-term memory is used to remember information for years.) The first list only had 6 items. The second list had 11 items. But it was more than twice as hard to memorize. Why? It had gone over the limit.

Now let's look at an example that is a bit closer to math.

Here is a rule: When you see a cow, hit it with a frog.

Easy to memorize. Easy to understand. You might even find yourself remembering this "formula" several weeks from now.

Here is another formula:

When you see a ztyq, hit it with a tfgh. (Note: A ztyq is just a cow missing a leg. A tfgh is a frog with more that seven spots on his back.)

If you focus, you will be able to memorize this formula and this explanation for a few minutes. But you will probably forget it by tomorrow. There are two reasons for this. First, there are more items of information. Secondly, picturing this requires a bit more work.

If you have ever struggled with math, the feeling you get from the above "formula" may be familiar. Now try this:

When you see a mtyq that is lacking a tfgh, hit it with a mrtg. (A mtyq is a ftzz or a qoiu. A qoiu is a frog without feet. A mtyq is half a dandelion. The definition of a tfgh is given above. An fter is a half of *a rofr, which is a cow's left hoof.)*

This formula is extremely difficult to follow. Few people even bother to read the formula the whole way through, and those that do forget it quickly.

What does this have to do with math? Look at the following math problem:

Paint costs $3 for enough paint for one square foot. Fred wants to paint a rectangular wall that is 4 yards wide by 5 yards long. How much will it cost to buy enough paint for 3 coats?

If you know that the area of a rectangle is length times width, and that a yard is 3 feet, this problem should not cause cognitive overload. But if you do not have the facts and formulas memorized, you end up with:

Paint costs $3 for enough paint for one square foot. Fred wants to paint a rectangular wall that is 4 yards wide by 5 yards long. How much will it cost to buy enough paint for 3 coats? (The area of a rectangle is length times width. A yard is three feet.).

It looks familiar, right? We have not even come close to cognitive overload, but there is more to juggle now. Because the student must juggle the information in the problem and unfamiliar formulas, he is not able to focus exclusively on solving the problem.

Now look at this problem:

Fred wants to paint a can red. The can is a cylinder with height 20 *inches and radius 10 inches. He wants to cover the sides of the can with three coats of paint and the top with four coats of paint. Paint costs 10 cents for enough to cover a square inch. How much will it cost, in dollars, for enough paint to paint the can?*

This problem is a bit more complicated, but it is still only a prealgebra problem. Now look at what a student who does not know the formulas must juggle:

Fred wants to paint a can red. The can is a cylinder with height 20 inches and diameter 10 inches. He wants to cover the sides of the can with three coats of paint and the top with four coats of paint. Paint costs 10 cents for enough to cover a square inch. How much will it cost, in dollars, for enough paint to paint the can. (The top and bottom are circles; the area of a circle is π**radius*^{2}*. The radius is half the diameter. The lateral surface area is the circumference times the height. The circumference is* π**diameter.)*

Of course, in a real problem, the relevant information would not be neatly written in parentheses after the problem. The student would have to look it up, or ask a parent or teacher. He would not only have to juggle the information, but also keep it all together while he got it from different sources. He would have very few cognitive resources available to analyze and solve the problem, because his mind would be too occupied keeping the formulas straight. He would have little chance of getting the problem right; on a test, he would just hope for partial credit.

This problem would be given in a prealgebra class, usually to seventh or eighth graders. And yet many high school seniors would struggle with this problem. In fact, many adults would struggle with this problem. But the problem is not actually difficult; it just tends to create cognitive overload.

When a student hits cognitive overload, the signs are usually easy to see. He becomes visibly frustrated. He may act out emotionally, by yelling, crying, or swearing. This is often viewed as a deep-seated behavioral problem, but it often is not. The student is faced with an impossible situation that may seem pointless. How would you react if you woke up tomorrow morning locked in a cage, for no apparent reason?

Other students may withdraw, seeming as if they are somewhere else. Their faces may stop showing any expression, and they may show little reaction to instructions or questions. Teachers and parents often mistakenly conclude that such students are stupid. In reality, they are withdrawing from an incomprehensible situation.

Some students may write something completely random on the page, or blurt out a formula that has nothing to do with the problem. For example, they might just say "quadratic formula?" or "Pythagorean theorem?" They might even just guess a random number. A student who does this does not believe that his random utterance is the correct answer. In desperation, he just says something, knowing full well it is wrong.

If a student has been struggling for a long time, you are dealing with an even bigger problem. Remember this?

When you see a cow, hit it with a frog.

You know what a cow is, and you know what a frog is. So it is easy for you to understand the above rule.

With the cylinder problem above, the really struggling student sees something more akin to this:

When you see a faquat, hit it with a potwu.

Why? He might not know what a cylinder is. The phrase "lateral surface area" might as well be written in Babylonian. Diameter? Radius? Because he cannot picture the problem properly, the steps he must take are a meaningless series of commands. If by some miracle he remembers them for a quiz, he is sure to forget them by the exam. He is dealing with foreign concepts that he can not picture.

To picture the problem, he must store the following information in working memory:

1. What a cylinder is
2. What a circumference is
3. The formula for the circumference
4. The formula for area of a circle
5. What a radius is
6. What a diameter is
    6a. The relationship between radius and diameter
7. The formula for the lateral surface area (which is really just the area of a rectangle)

The student has hit seven before even starting the problem. His working memory is full, and the problem has not even begun! He has nowhere to store the information for the problem (what the height is, what the cost is, etc.)..

Additionally, when a student's cognitive resources are being fully used, the student is less able to check for random errors. The rate at which he makes careless mistakes goes up dramatically. In fact, a high number of careless mistakes is one of the signs that tell me that a student's cognitive resources are being overstretched during the problem-solving process.

This is a prealgebra problem. The difficulty increases as the student enters algebra, or moves up to calculus.

HOW THE ASIAN SYSTEM ADDRESSES COGNITIVE OVERLOAD

We discussed how working memory can hold about 7 pieces of information at one time. But you know more than seven facts. You know more than 7000 facts, for that matter.

That information is stored in long-term memory. The Asian system helps students store information in their long-term memory in such a way that it is readily accessible. To be more precise, the Asian system forces students to store information in their long-term memory, and to have it ready for use.

The Asian system is fantastically effective, and extremely simple. As soon as the child is able to talk, math training begins. The child is constantly taught to memorize math facts and drilled daily on the facts. He is quizzed constantly on his multiplication tables. He is quizzed constantly on formulas (e.g. area of a triangle, circumference of a circle, quadratic formula, etc.).

He is repeatedly given specific problem types until he can do them in a few seconds. For example, he might be asked to find the surface area of a cylinder every day. After a few weeks, he can find the surface area of a cylinder with incredible speed. With the constant drilling, the information is always readily accessible and is stored in long-term memory.

The student does not need to have any amazing innate intelligence. His intelligence can be just average. For that matter, it can be below average.

The results speak for themselves, but let's look at how this student analyzes the above problem. Remember, the formulas are so ingrained into his mind that he barely needs to think about them to use them. He has done problems like this one so many times that the process has become virtually automatic.

Here is the problem mentioned in the last section:

Fred wants to paint a can red. The can is a cylinder with height 20 inches and radius 10 inches. He wants to cover the sides of the can with three coats of paint and the top with four coats of paint. Paint costs 10 cents for enough to cover a square inch. How much will it cost, in dollars, for enough paint to paint the can.

Here is the Asian student's way of thinking about it:

Find the top area and multiply by 4. Find the lateral surface area and multiply by 3. Add the two areas, and then multiply the sum by 10 to get number of cents. Divide by 100 to get the number of dollars.

The correct mathematical steps are:

$\pi^*(10)^2 = 100\pi$ is the top area. Multiply this by 4 to get 400π. The lateral surface area is $2\pi(10)(20) = 400\pi$.
Multiply this by 3 to get 1200π. Add those two numbers together to get $400\pi + 1200\pi = 1600\pi$. Multiply this number by 10 to get

the number of cents, which is 16000π cents. Divide this by 100 to get 160π dollars. Note that π equals approximately 3.14.

COGNITIVE OVERLOAD IN ARITHMETIC AND ALGEBRA

We have seen how cognitive overload can be an issue when solving word problems. What about regular arithmetic and algebra problems?

Look at this problem:

$$\begin{array}{r} 45 \\ \underline{\times\ 37} \end{array}$$

Most adults would find this problem fairly straightforward. But what if you had not memorized your multiplication tables? Then rather than starting out by doing 7x5 = 35, your first step would be to add 5+5+5+5+5+5+5, to get 35. You would then carry the 3, and do 7+7+7+7 (instead of 4*7) to get 28, and the process would continue like that. You might even forget what you were doing before you finished, and have to restart. In other words, you would reach cognitive overload.

The chance of making a careless mistake would be pretty high. You might even be tempted to do 45+45+45+45… (37 times) rather than the step by step multiplication.

How about a harder problem?

$$\begin{array}{r} 4563 \\ \underline{\times\ 7452} \end{array}$$

A bit tougher. But if you did not know the multiplication tables, it would be incredibly difficult.

Keep imagining that you did not know the multiplication tables. Do this:

$$1/7 + 5/42$$

It is getting harder, right? If you do not know multiplication tables, it is hard to get started with this one.

Now let's move to algebra:

$$3/a + 4/(a+3)$$

This problem has nothing to do with multiplication tables. But a person who had not memorized multiplication tables would never have really understood how to add fractions. That person would never be able to understand how to do this problem. He would probably memorize some way of doing the problem in the short term, and forget it by the time the exam comes around.

The above problem is a beginner level algebra problem. More advanced problems would require the student to solve similar problems as just one part of a multi-step problem. As the difficulty increases, the problems become impossible for the child to attempt at all. The child has "slipped through the cracks." There is no way for the child to do the problem without cognitive overload. To understand and solve the problem, he would have to hold years' worth of material in his working memory, which is impossible. Alternatively, he would have to actually learn several years of math before attempting the problem.

Because the Asian system focuses on long-term memorization of basic math facts, those trained with this system never face cognitive overload on these types of math problems. In fact, the Asian system takes it one step further. Not only does the system force students to memorize multiplication tables, etc., it constantly drills students on basic problem types. The student would not just find it easy to figure out how to do the above algebra problem. He would not need to "figure it out" in the first place. He would have practiced similar problems so many times that the process would be virtually automatic; it would be no more difficult than walking.

DISTANCING

Remember this?
"When you see a cow, hit it with a frog."

That is a lot easier to remember than
"When you see a terqp, hit it with a srato."

Why? The two statements are equally complex. However, the first statement means something to you, while the second statement means nothing. You cannot visualize it, or make sense of it beyond committing it to rote. You can memorize it, but you do not really know what you have memorized. Your mind distances itself from the information. You might memorize the information, but it will never be fully incorporated into your understanding.

Let's see how this applies to math. Student A is a strong math student. Student F is a weak math student. Both are given the following formula:

Area of a circle is π**radius squared, where* π *=3.14159…*

They are both given the following problem:

The radius of a circle is 6. Find the area, in terms of π.

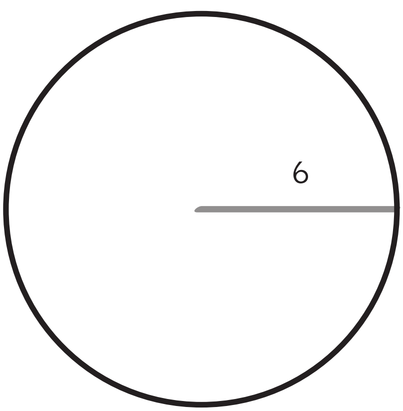
Both students do the following:

Area=radius^{2*} π

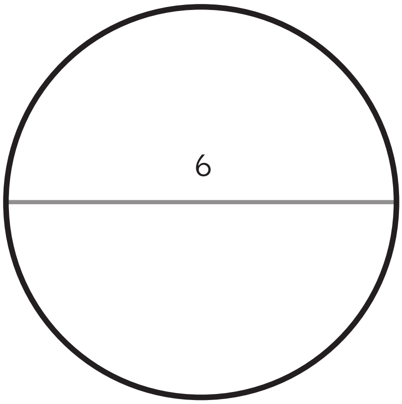
Area = 6^{2*} π

Area = 36 π

Internally, however, they did something completely different. Student A (the strong student) pictured the problem. Before he began, he visualized the following:



He knows what a radius is, and understands what he is doing as he calculates the area. If he is instead given a problem that says the diameter is 6 and asked to find the area, he will visualize this:



He will instantly see that the radius is 3, and will then solve the problem.

Student F (the weak student), did something very different. At some level, he thought "I don't know what a radius is, and I don't want to know. All I need to know is that if I am given a radius, I should multiply the radius by the radius, and then multiply by π."

I call this phenomenon "distancing", because the student distances himself from the concept. He keeps the information outside of his perception of the world. If the strong math student sees a pizza, he recognizes that it has a radius, and can picture the radius. The weak math student thinks of the word "radius" as descriptive only in his math class. He does not even consider that every circle he ever sees has a radius.

If Student F is given a problem that gives him the diameter instead of the radius, he is confused. Often, weak students end up just memorizing that the radius is the diameter divided by two, without visualizing the relationship. Not only does this further distance the student from the concept, it also gives him an extra item of information to memorize.

As math becomes increasingly complex, the student becomes increasingly distanced. He memorizes formulas by rote for each test, and comes close to failing each exam. The formulas mean nothing to him; he memorizes them by rote, just as you might remember

"When you see a terqp, hit it with a srato."

HOW THE ASIAN SYSTEM ADDRESSES DISTANCING

In the phenomenon known as distancing, the student uses temporary rote memorization to get by on math quizzes. The previous section explained why this is problematic.

At the same time, the Asian system is built on rote memorization. Students are often taught to memorize formulas well before they can understand them. A nine year old might memorize the quadratic formula well before he understands what the formula is used for. It would seem that this is a guaranteed way to create distancing.

However, it does the opposite. The information becomes fully integrated into the student's understanding.

Remember this?

"When you see a terqp, hit it with a srato."

It still means nothing, but it is starting to become more familiar.

How does the mind decide what information to incorporate and what information to "distance"? One consideration is the relevance of the information. Relevant information is easier to incorporate. For example, you might find the information in this book easier to incorporate than the information in a tax manual from the 1950s.

A second consideration is how interesting the information is. The pufferfish, which contains a highly potent neurotoxin, is considered a delicacy in Japan. Because of the potential dangers of eating pufferfish, it is the only delicacy forbidden to the Emperor of Japan. This fact is interesting, so it is easy to incorporate into permanent memory.

On the other hand, consider the following. The Black-Tailed Rattlesnake produces a highly toxic venom that is dangerous to man. This fact is less interesting, so it is easier to forget (even though it is simpler).

The next consideration is complexity. More complex information is more difficult to incorporate into memory. For example, it is easier to memorize "When you see a cow, hit it with a frog" than to memorize "When you see a cow, hit it with a frog, unless the cow is speckled, in which case hit it with a frog only if it is a weekend."

The final consideration is familiarity. Facts that are in some way familiar are easier to incorporate into memory than facts that are distanced. Take this for example:

"When you see a terqp, hit it with a srato."

It is still weird, but it is starting to sink in. If you saw this every day, and were constantly quizzed on it, you would have it memorized. And if one day you were given an explanation as to what it meant, it would be instantaneously incorporated into your understanding and long-term memory.

And that is how the Asian system prevents distancing. It prepares the mind to incorporate important information into its permanent understanding. By the time the child learns how to use the quadratic formula, it is so deeply ingrained into his memory that it becomes immediately incorporated into his mathematical understanding.

By constantly drilling information, the Asian system familiarizes the student with the information, preventing him from distancing himself from it. This prepares the student to instantaneously incorporate information into his permanent understanding as soon as he learns how to use the formula. Thus, paradoxically, by using rote memorization as a training tool, the Asian system prevents students from relying on short-term rote memorization, and instead makes them incorporate the information into their permanent understanding.

HIERARCHIZATION, AND ERRORS IN HIERARCHIZATION

One of the biggest differences between strong math students and weak math students is the way in which they hierarchize information (i.e., how they rank information in terms of its importance).

Strong math students generally hierarchize different concepts and formulas. For example, a strong math student recognizes the quadratic formula as an extremely important formula, and is able to easily recall it at any time. However, a formula of less importance, such as Descartes' rule of signs, gets less priority. The strong math student may be able to recall the rule after thinking about it for a few seconds, or he may even have to look it up. Similarly, the strong math student will instantly know how to factor

a^2-b^2 (the answer is (a-b)(a+b).)

However, he may need to take some time to remember how to factor

a^3-b^3 (the answer is $(a-b)(a^2+ab+b^2)$.)

Even extremely strong math students, who are able to quickly do either problem, have the processes hierarchized. For example, the extremely strong math student will be able to do the first problem in about a tenth of a second, but may take up to two seconds to do the second problem. In other words, the second math problem will take 20 times as long as the first.

The weak math student, on the other hand, often hierarchizes the information incorrectly or not at all. In the first case, he gives undue weight to concepts of less importance, and insufficient weight to important concepts. In the latter case, he gives all concepts and formulas about equal weight. While he may actually remember unimportant concepts faster than strong math students, his skills with the important concepts are underdeveloped. While at the end of each year the strong math students remember about ten key concepts and are able to apply them flawlessly, weak math students have a vague and tenuous understanding of over a hundred math facts.

Students with ineffective hierarchization methods do very well on quizzes, poorly on tests, and horribly on exams. By the time they take the exam, their mind is so full of unimportant facts and formulas that they are completely overwhelmed.

These students often do extremely well in history classes, because of their capacity to remember large amounts of information for a few days or weeks. However, this very ability gets in their way in math classes. Because they can remember a huge amount of information for a period of weeks, they have less need to hierarchize information. Thus, they learn the information in an unhierarchized manner, giving the same weight to vital and unimportant information, and forget the important concepts along with the unimportant ones.

When taught concepts that are completely unimportant, strong math students will sometimes perform worse than weak math students. They instinctively recognize the information as irrelevant, and find it almost impossible to memorize the formulas.

HOW THE ASIAN SYSTEM HANDLES HIERARCHIZATION

American math classes and textbooks rarely employ effective hierarchization. Instead, most courses and books present essential information mixed in with a large amount of unimportant filler information. One week a student may study factoring, which is extremely important. The next week the class may focus on stem and leaf plots, which are relatively unimportant (a stem and leaf plot is a rudimentary way to organize statistical data.) The strong math students develop an intuitive ability to thoroughly understand the important information, and deemphasize the unimportant. Weak math students do not develop this ability, and thus struggle through math classes.

On the other hand, the Asian system does not rely on a student's innate ability to hierarchize information. Instead, the information is presented in a hierarchized manner.

As previously discussed, the Asian system drills students constantly. However, it does not drill students randomly. Students practice adding fractions. Students are quizzed on the quadratic formula. Students are quizzed on basic derivatives and integrals. Students are quizzed on the definition of sine, cosine, and tangent. Remember that they are not just quizzed right before a test on the subject. They are quizzed all the time. A ten year old student may be quizzed on the definition of sine. His first school test that asks the definition of sine may not be for another five years.

However, students are not quizzed on information of secondary importance. Students are not given stem and leaf plot practice problems, except right before a school test on the subject. Students are not quizzed on Descartes' Rule of Signs (a formula often taught as part of precalculus). It is not that these concepts have no importance; they just do not have the fundamental importance of skills like multiplying fractions or factoring.

The Asian system does not wait for students to hierarchize information. Instead, by strongly emphasizing important information, the Asian system ensures that students develop appropriate hierarchies.

Over time, the Asian method actually helps students develop their own hierarchization skills. Because important information is strongly emphasized, they begin to develop an intuitive understanding of what type of information is important in math. As they study additional topics in math, they learn to rely increasingly on their own ability to hierarchize information.

HOW THE ASIAN SYSTEM ENSURES UNDERSTANDING

We have seen how the Asian system enables students to solve problems efficiently. We have seen how the Asian system prevents distancing and ensures correct hierarchization. But what about understanding? How does the Asian system ensure that students actually understand what they are doing?

As you become more and more familiar with something, your mind becomes more comfortable exploring it. In math, students are more comfortable exploring familiar topics than unfamiliar ones. As they explore the topics consciously and unconsciously, their understanding of the topic automatically increases.

The Asian system, then, does not always teach students to understand the topic directly, at least initially. Rather than giving elaborate explanations, the Asian system focuses on making sure students are so familiar with the material that they are comfortable exploring it on their own. The Asian system does eventually give explanations, but they often come after the student has mastered the mechanics. For example, students are taught the mechanics of multiplying fractions first, and then given an explanation about why the method works.

By the time a student gets an explanation, he is already familiar with the mechanics. This allows him to dedicate his full cognitive resources towards understanding the problem, instead of having to split his attention between understanding the concepts and learning the mechanics.

DOES THE ASIAN SYSTEM TURN STUDENTS INTO DRONES?

In the opening chapter, I discussed why students should study math. I explained that math develops the mind, teaches students to analyze and solve new problems, etc. But on the surface, the Asian system seems to do none of them. It seems to turn students into unthinking automatons. Yes, they are able to do specific math problems quickly, but for most people math is not important for itself. It is only important because of the ways in which it develops the mind. The Asian system trains the mind, and drills the mind. But does it develop the mind? Does it increase intelligence? Or does it just turn students into drones who can quickly do specific types of problems?

The final and most important piece of the Asian system addresses this concern. Drilling is essential. Practice is essential. Memorization is essential. But it is not enough. The student also needs challenging problems.

Challenging problems are the ones that take anywhere from 20 minutes to a week to figure out. These are the problems that force students to put their knowledge together in new ways and to really stretch their mind to figure out new problems. This process makes students smarter, not just better at math.

It is easy to make up challenging problems for younger kids: just give them slightly more advanced problems. For example, if they already know how to multiply single digit numbers, give them a two digit multiplication problem and have them struggle with it. Whether or not they solve it does not really matter. As long as they really struggle, their minds will be developing.

If you are good at math, you will probably be able to make challenging problems for older kids as well. But if you are not, you can use other sources for math problems, such as SAT I and SAT II math practice tests. These can be found in practice books for these tests, which can be found in almost any bookstore. The problems towards the end of a section are usually the hardest. For example, if a math section on the SAT has 25 questions, questions 23, 24, and 25 will usually be the hardest.

USING THE ASIAN SYSTEM

Here are a few guidelines that can help you get your child started with the Asian system.

1. Set aside a specific time every day for math. The standard is 1 hour per day, every day, for math practice. This is in addition to any school math homework, and runs year round, including vacations.

2. Traditionally, in addition to the daily hour of math preparation, parents randomly quiz their children on math facts and math problems. This can take place on car rides, during meals, etc.

3. Materials: You can use math textbooks and math workbooks. If you have excellent math skills, I recommend using the University of Chicago School Mathematics Project materials, commonly known as "Chicago Math." It is an "expert system" in that you really have to understand math well to be able to understand and use the system effectively.

4. Fanatic dedication. All children (even Asian children) initially resist the Asian system. They will point out that their friends do not have to do the extra math work, and will do whatever they can do get out of it. Make sure they do the extra training. Your child may never enjoy it, but he will quickly see the benefits.

5. Constant Review: During the process, make sure to repeatedly revisit older topics.

6. Focus on the basics. The hour a day should be spent on major problem types, such as adding fractions, rather than on

minor problem types, like stem and leaf plots.

7. More is better. Two hours a day is better than one hour a day. Three is even better.

8. Repeatedly remind yout child that math develops the brain, and that by doing the extra math they are becoming smarter than their peers. Many of my math students voluntarily put in several hours of math training per week in addition to homework and tutoring sessions, because they know that the training is something that benefits them, not an obligation to someone else.

You should start using the Asian system today. If your child can talk, start the process. And it is never too late. You can start using the Asian system with a child who is 17, or with an adult for that matter. In fact, many of my older students voluntarily put themselves through the Asian system, with excellent results, as do some of my younger students. However, for most younger students, you will need to make them do it. They will not like it, but it will make them much more successful at math and life.

USING COGNITIVE OVERLOAD

The Asian system helps students avoid cognitive overload. Interestingly, in some cases you can actually use cognitive overload as a powerful cognitive incentive.

Suppose a child insists on multiplying by adding, rather than by using memorized multiplication tables. For example, the child will do 19*6 by adding up 6+6+6+6+6+6+6+6+6+6+6+6+6+6+6+6

+6+6+6. This method is slow, arduous, inefficient, and painful to watch. Often, it is almost impossible to convince the child to use memorized math facts.

Of course the child cannot do a problem like 43*96 without using memorized math facts. But many parents and teachers hesitate to give the student a problem like that, if the student is still using the aforementioned inefficient method. They feel that if they give such a student a problem like 43*96, the student will feel totally overwhelmed.

And they are right! The student will feel totally overwhelmed. And that unpleasant feeling gives the student a powerful incentive to switch to the more effective method.

What I usually do is give the student a mix of easy and hard problems. Hard problems give the student the incentive to learn the more effective method, which they can then practice on both the hard and the easy problems.

3: SELF-PERCEPTION AND POLARIZATION

SITUATION 1:

You just started seventh grade, and it is the first math class of the year. The teacher explains something, and you notice that others in the class seem to understand the concepts faster than you do. You are able to solve the problems, but it takes you a bit longer than the rest. By the end of the class, you are starting to think that you just might be bad at math.

The next day confirms it, as the other students seem to race ahead of you. In reality, they are at best 5% faster, but from your perspective it seems like they are at least ten times better at math than you are. At this point (two days into the year), you are already pretty sure that you are bad at math. By the end of the week, you are absolutely certain. In fact, being bad at math soon becomes part of your identity. When the teacher gives a challenging problem, you do not really try that hard to solve it. That is just not who you are. You are the kid who struggles with math; what chance do you have of solving a hard problem?

You do your homework, but if you cannot get a problem, you are not too concerned. Athletes play sports. Rockstars sing. Your role in life is to get math problems wrong.

Every so often, you get some evidence that does not fit your theory. You get a 9/10 on a quiz that everyone else in the class fails. But you and the rest of the class are now so sure that you are bad at math that you decide that the quiz is flawed. Everyone laughs about it. How did the bad math student do better than everyone else on the quiz? Even your parents think it is kind of funny.

From time to time you get a math teacher that tells you that you can be an excellent math student, and that there is nothing wrong with your abilities. Sure, he is something of an authority. But you have years of experience, the opinions of other teachers, and the opinions of your friends on the opposite side. Obviously, you assume that the heretical math teacher is wrong and that every other person on Earth is right.

SITUATION 2:

You just started seventh grade and it is the first math class of the year. The teacher explains something to the class, but it is something that your mom taught you over the summer. You already know it, so you are the first one to understand the concept. Everyone around you notices, and the teacher marks you as one of the strong math students. In fact, by the end of the class, you are officially one of the excellent math students. That is how the teacher and every other student has started to see you, and it is how you are starting to see yourself. By the end of the week, it has become part of your identity.

From time to time, you come across a homework problem you cannot immediately figure out. But what if someone else in the class figures it out? Athletes play sports. Rockstars sing. Your role in life is to get more math problems right than any other student, and you will fill that role no matter what it takes. It may take you five hours, but you will figure the problem out.

Every so often, you get some evidence that does not fit. On one quiz, you get a 5/10, and everyone else gets at least an 8/10. The dumb kid gets a perfect score. Obviously, there was something wrong with the quiz. Everyone thinks it is funny, and the dumb kid announces to the world that he is now officially smarter than you. Much hilarity ensues, and even your parents think it is kind of funny. Your dad tells you not to make a habit of it, but it is more of a joke than a warning. Of course you will not make a habit of it. That is not who you are.

Two things happened in each situation. The first was that the child quickly developed a perception of his own ability by comparing himself to the rest of his class. Note that self-perception was determined by ranking, not by measuring. The struggling child did not measure how far behind the rest of the class he was; all that mattered was that he was behind. He was in last place. Whether the child finished ten seconds behind the rest or ten minutes, he was at the bottom.

Secondly, in each situation the class sorted itself into a hierarchy, and this hierarchy reinforced the child's self-perception. In any group, various hierarchies will be established. The tallest kid in a group is the tall kid, even if he is only an inch taller than the rest. The fattest kid is the fat kid, even if he is not really that fat. The slowest kid is the slow kid, even if he is only slightly slower than the rest. The smartest kid is the smart kid, even if he is only slightly smarter than the rest. Small differences become exaggerated, and the group becomes polarized.

The good news is, of course, that to make a student see himself as a smart kid, he does not have to be that far ahead of his peers. He only needs to be slightly ahead; once he and the rest of the class perceive him as a smart math student, self-perception and polarization will become the child's powerful allies.

The simplest way to take advantage of this is to use part of the summer vacation to cover the first few weeks of material. Find out what textbook the child will use in the coming year, and then go over the first few chapters. Your child will start the year slightly ahead of the pack, and will be much more likely to see himself as an excellent math student (as will his teacher and peers).

However, if everyone else in the class is doing the same, you will have to do more to keep ahead. Here is a secret: approximately 100% of Asian parents use this method. One of the biggest reasons that Asian American students do so well in math is that their parents make them spend part of their summer (usually an hour or two every day) doing math.

CHANGING SELF-PERCEPTION

Summer, winter, and spring vacations are the easiest times to change a student's self-perception, and his position in the academic hierarchy of his class. While his peers squander their vacations, your child can move forward. Thus, when school begins, he will start at the head of the curve.

Also, by making your child work during the vacations, you are making him see himself as the type of student that works through vacations. He accepts extra training as part of his life, and that extra training will carry him far.

BEST OF THE WORST OR WORST OF THE BEST

Sometimes, parents who are considering different schools ask whether it is better to put the child in a more competitive school in which he will struggle or in an easier school in which he would lead the pack.

First, those are never the only two options. By training extra, the student can lead the pack in the more competitive school.

But let's pretend that those were the only two options. The child can either be the best student in an easy school or a mediocre student in a difficult school. Which option is better?

As we discuss the importance of self-perception, it is important to remember that there is such a thing as reality. The student who leads the pack in an easy school will see himself as a strong math student, and will usually be motivated to study hard. But he will lack the rigorous training that the tougher school can offer. Simply put, he just will not learn as much. He will not become as smart. And when he competes in the outside world, he will not be as able to succeed.

The person who finishes last place in an Olympic race is still a world class athlete. A student who struggles in a competitive school is still miles ahead of a student who excels in an easy school. Sure, the student in the harder school will struggle. But when he competes against those who lacked rigorous educations, he will easily surpass them.

4: INCENTIVE AND STRUGGLE: The Art of Developing the Mind

When taught right, math builds the mind in the way that lifting weights builds the muscles. But not all methods of teaching math do this equally; in fact, some of the more recently adopted methods of teaching math actually do the reverse. Not only do these methods fail to build cognitive skills, but they actually cause skills that the student has already built to atrophy.

Three things cause cognitive skills to develop. The first is age. Even with the worst education available, human biology will make a sixteen year old more intelligent than a two year old.

The second thing that causes cognitive skills to develop is exposure. Children who are exposed to interesting ideas and problem types can freely stretch their minds and explore new modes of thought. As a simple example, a child who plays with a Rubik's cube may develop a surer sense of three-dimensional reasoning.

The third consideration is incentive. A child who plays with a Rubik's cube may develop the foundations for strong spatial reasoning; however, without a strong incentive, he may never push his mind to the limit. If he cannot figure out how to solve the Rubik's cube, he will probably just give up.

For the mind to have the incentive to develop, two things are necessary. First, it must encounter a problem that it is unable to do; the process of figuring out how to solve this initially unsolvable problem causes the mind to develop. If a student is only given problems at his current ability level, what incentive does the mind have to improve? Just as lifting a half-pound weight will not make a person physically stronger, doing an easy math problem will not make a person mentally stronger.

Parents and teachers of gifted students often overlook this, and just allow them to work at a comfortable pace. The result is that the gifted students never get the opportunity to realize their full potential. Like natural athletes who never train hard, they end up squandering their innate talents.

No matter how smart a student is, he must be given some challenging math problems that he is initially unable to do. If he can solve the problem in five minutes, it is not a real challenging problem. An appropriate challenging problem should take anywhere from twenty minutes to a week to solve.

Once you have a sufficiently challenging problem, the next thing you need is incentive. If the student has no incentive to figure out a difficult problem, he will simply walk away. However, by understanding what motivates your child, you can design the right kinds of incentives. The following list includes some common motivators:

1. Desire to impress. If your child wants to impress you, and is willing to struggle through a difficult problem to do so, all you have to do is give him verbal approval when he solves a challenging problem.

2. Desire for self improvement. At some level, everyone wants to be better at everything. Everyone would like to be smarter and stronger. Sometimes this desire can get buried under other desires, but it never disappears. Tapping into this desire is the strongest method that I know of, and I favor this method above all others.

3. Material incentives. Parents may try to use money or toys as an incentive to work hard (for example, a student may be given $20 or a video game for every A he earns.) As discussed later in this book, this method is almost always completely ineffective.

4. Laziness. Using laziness is another one of my favorite ways to motivate students. Everyone likes to avoid work. If figuring out one particularly difficult problem will allow the student to avoid twenty hours of math work, he will put his full abilities into solving the problem.

Many parents unthinkingly do the reverse. The better the student does, the more work he ends up doing. Every correctly solved problem is rewarded with a more difficult problem. Students who are both clever and lazy often figure this out, and deliberately get problems wrong in order to avoid doing extra work. This phenomenon and the way to address it are discussed in depth later in Chapter 12.

Once you have a challenging problem and have given the student a strong incentive, all you have to do is sit back and watch him struggle. Be patient. Do not explain how to do the problem or give excessive hints. Stay completely calm, and wait for him to figure out how to solve the problem. At most, give one hint every five minutes.

Every second your child struggles with the problem, he is becoming smarter. His cognitive abilities are increasing bit by bit. If you can consistently create struggle, you will slowly but surely improve your child's reasoning skills.

When I watch a child struggle with a math problem, I imagine a meter at a gas station, with the numbers rapidly moving up. Every split second that goes by, the number increases. I imagine that this number corresponds to the child's intelligence. Every second I let the child struggle, his intelligence is increasing.

Give the student time with each problem. Struggling with one problem for several hours benefits the student much more than sailing through ten problems in ten minutes.

As you watch your child struggle, remember that the cognitive skills developed are not just good for math. They are the skills that allow a person to analyze and approach any kind of situation.

THE PATH OF LEAST RESISTANCE

Suppose you want to teach your child how to add fractions. Let's look at a few hypothetical situations.

1. You teach the child to add fractions by hand, and do not teach the child how to add fractions with a calculator. For all future problems, he must add fractions by hand, and is not allowed to use a calculator.
2. You teach the child to add fractions by hand first. Once he is able to add fractions by hand, you then teach him how to add fractions with a calculator. For all future problems he has the choice of adding fractions by hand or using a calculator.

3. You teach the child to add fractions by hand and how to add fractions with a calculator at the same time. For all future problems he has the choice of adding fractions by hand or using a calculator.

4. You teach the child to add fractions with a calculator only. You do not teach him how to add fractions by hand. For all future problems, he must use a calculator.

Which is the best option? At first, option 2 looks the best. In this situation, the student learns to add fractions by hand and also becomes proficient at adding fractions with a calculator. This option seems to combine the best of both worlds.

Most people who choose option 2 will say that option 1 is the second best, option 3 is the third best, and that option 4 is outrageously irresponsible. However, for all practical purposes, options 2, 3, and 4 are identical.

Let's look at option 4 first. The child was never taught to add fractions by hand, and he has no incentive to add fractions by hand, since he can always just use a calculator. Thus, he will never learn to add fractions by hand.

In option 3, the child can always choose either to add fractions by hand or with a calculator. Both methods will give him the answer, but using the calculator is easier. Thus, the student will probably always use the calculator. Even if he initially learns to add fractions by hand, this skill will atrophy through chronic calculator use.

Finally, let's take a look at option 2. At first, the student is able to add fractions by hand. However, he is then always given the option to add fractions with a calculator. He will most likely always use the calculator, and never add fractions by hand. This will allow his skill